Volatility, Non-Randomness or Non-Linearity: What Drives Portfolio Returns In Times of Stress and Dislocation?

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1. Introduction

This article considers what underlying exposures drive portfolio returns in periods of extreme market stress and dislocation. Those investors who were shocked in the recent financial crisis by a perceived failure of diversification, and are now keen to avoid a repeat of that painful experience, need to identify and correct unintended risk gaps and biases in their portfolios.

One way to look at the problem, which is increasingly gaining currency, is to consider it from a risk contribution perspective: it can be shown that even very diverse multi-asset class portfolios are typically dominated by broad equity risk, which can account for more than 80% of total risk. Consequently, the ongoing debates about new approaches to asset allocation and tail risk hedging are essentially trying to address the same problem, i.e., how one can limit or reshape a typical institutional investor’s exposure to broad equity risk to make the portfolio more robust.

In this paper we take a slightly different angle, drawing on academic research and relevant practitioner experience in the areas of hedge fund investing and options trading. Specifically, we consider three types of underlying return drivers: (1) exposure to volatility, (2) exposure to non-randomness in returns, and (3) exposure to non-linearity in returns. We compare and contrast these three schools of thought and conclude that it is non-linearity, or, more specifically, convexity, that appears to offer the most comprehensive and consistent framework for finding solutions to mitigate portfolio fragility. While this paper outlines the theoretical case for convexity, in our next paper we shall focus on empirical analysis, as we attempt to build a universe of convex strategies and to develop the concept of a “convexity completion portfolio.”

Before we proceed with our analysis, given that we are focusing on investors with the proverbial deep pockets and very long time horizons – sovereign wealth funds, endowments, foundations, and pension plans – we need to acknowledge two important caveats.

Caveat #1: trading growth optimality for robustness

Despite the multiple and varied alternative approaches to asset allocation and tail risk hedging, on a fundamental level, there is a common thread to all of them which is critical for a long-term investor to consider: they all represent a trade-off between growth optimality and robustness. Each alternative asset allocation solution effectively forces the investor to give up a portion of higher long-run expected returns from equities and other growth assets by rebalancing the portfolio towards safer assets with more pedestrian long-run expected returns. Similarly, each tail risk hedging solution inevitably introduces a drag on portfolio returns due to the costs of hedging. While these costs can be managed and potentially minimized, it is unlikely that they can be reliably and permanently eliminated.1

Once the nature of this trade-off is fully understood and communicated to fund beneficiaries, stakeholders and other interested parties, the policymakers and executives in charge need to be absolutely clear and brutally honest in answering the following two questions: Is there a genuine economic or financial reason that compels us to reshape our portfolio so that it is robust and resilient in the face of another major crisis, or are we being
forced to unnecessarily give up growth optimality to satisfy political and reputational anxieties? If it is the latter, then before one even thinks of tinkering with the current portfolio, maybe the correct solution lies in the areas of institutional design, governance, public relations and financial education. But if there is a genuine economic or financial reason to seek protection against occasional steep declines in portfolio value, then the fiduciary focus must be on finding the most effective and reliable solution that would be least disruptive to long-term growth optimality.

Caveat # 2: growth optimality of equity-centric portfolios
Reliance by very long-term institutional investors on equity as the main engine of growth is both conceptually sound and empirically justified. It can also be shown mathematically that mean-variance efficient portfolios necessarily contain on the efficient frontier a growth-optimal portfolio as defined by the Kelly criterion. So if you are a genuinely unconstrained investor with a very long-term investment horizon and you want to manage wealth in the most growth-optimal way, irrespective of interim volatility and illiquidity, the best recommendation may well be to change absolutely nothing in your traditional approach to asset allocation.

Of course, one potential problem with this widespread and well-documented investment belief is that our perception of equity as an asset class is conditioned almost entirely on our experience of the good years, with solid and relatively uninterrupted economic growth throughout most of the developed world, especially in the post-war period. The last decades of the Great Depression and of the more recent variety in Japan are viewed as an aberration. But what if the world today is on the verge of a similar multi-decade experience? This is a different kind of tail risk, much more relevant to unconstrained long-term investors, yet not directly hedgeable in capital markets. This tail risk does not bode well for a traditional equity-centric portfolio. So while we continue to base our discussion on the premise that an equity centric-portfolio is growth-optimal, we need to be mindful of the low-probability, high-impact nature of depression economics.

2. Three schools of thought
Now let us tum to the main focus of the paper and consider in turn the three schools of thought which attempt to describe and explain the underlying drivers of portfolio returns in times of extreme market stress and dislocation.

Anyone who has worked in the hedge fund industry during the last 10-15 years, or closely followed different investment strategies – looking at how they performed in different macroeconomic environments and how they reacted to various shocks – would have at least some familiarity with three overlapping, yet distinct schools of thought that attempt to classify funds based on their exposures to the following return drivers:

1. Levels and/or changes in volatility (i.e., long vol versus short vol)
2. Non-randomness in returns (i.e., trending versus mean reversion)
3. Non-linearity in returns (i.e., convex versus concave)

While these three stylized frameworks are usually invoked in discussions of hedge funds, there is no reason why we could not apply them more broadly to a more traditional multi-asset class portfolio. In fact, the concept of an institutional investor’s exposure to non-linear payoffs, as it was first laid out in Perold & Sharpe (1988), had nothing to do with hedge funds: it looked at convex and concave return patterns arising from various portfolio rebalancing rules.

Although the other two schools of thought appear to have their origins in hedge fund strategies and options markets, a fund manager who only invests in equity and bonds can still derive valuable insights by asking the
following questions: What happens to my portfolio if equity volatility spikes? What is my effective long or short volatility position? Do I need to mitigate it in the options market or should I simply rely on disciplined contrarian rebalancing in expectations of mean reversion? What exactly determines whether we are in a trending or mean-reverting environment, and how, if at all, is it linked to levels or changes in volatility? These are just some of the questions where applying the above typology of portfolio exposures may prove helpful in the context of traditional portfolio management.

In this white paper we focus our analysis primarily on hedge funds. We do this for three reasons. First, we believe it is sufficient for purposes of exposition and discussion of the overlaps and differences between the three schools of thought. Secondly, the main ideas and general principles identified in the context of a multi-strategy, multi-manager hedge fund allocation should be applicable more broadly across the entire portfolio, with one possible caveat of capacity. Thirdly, it is our strong belief that our hedge fund expertise, with a particular focus on global macro, is where we can offer most value to institutional investors.

2.1. Exposure to volatility
Analysis of hedge funds in terms of their long or short exposure to volatility has its roots in the seminal papers of William Fung and David Hsieh, who have done extensive work on identifying and describing systematic risk-factor exposures of different hedge fund strategies and how one might use this knowledge to replicate hedge fund return streams. Specifically, Fung and Hsieh (1997a, 1997b, and 2001) studied trend-following CTA strategies, which they managed to successfully model and replicate using portfolios of look-back straddles. As in plain vanilla straddles, these are combinations of call and put options, but the crucial difference with the look-back feature is that it grants the investor the right to buy the underlying instrument at the lowest price and to sell it at the highest price during the life of the options contracts. The authors showed that this payoff profile is effectively equivalent to that of a trend-following CTA manager.

Fung and Hsieh applied a similar type of analysis to other hedge fund strategies, as did other researchers, including Mitchell and Pulvino (2001), Lo (2001), Anson and Ho (2003), Agarwal and Naik (2004), and Jaeger (2008). Invariably, they showed that returns from most relative value or arbitrage-type strategies can be described as being equivalent to a short option position: earning steady and uncorrelated returns during normal times, but

![Exhibit 1 Payoff to Straddle](source: Author)
having significant left tail risks in times of market dislocation.

Conversely, trading strategies that benefit from strong market moves, such as CTAs and global macro, were described as being long optionality. This powerful insight caught on with market practitioners, who increasingly started to refer to different hedge fund strategies using options market terminology, namely, as being long vol or short vol.

This classification has become useful shorthand when referring to strategies that can be expected to do well or poorly in times of market stress. For purposes of our analysis, we are particularly interested in those strategies that do well in a crisis, as defined by a collapse in equity markets. Most academic research suggests that the majority of hedge funds are either linear in their risk relative to the equity market (e.g., long/short equity funds) or are effectively short a put option on the underlying equity market (e.g., relative-value or arbitrage funds), and as such would amplify rather than mitigate left tail risk. The all-important exceptions are global macro managers, both systematic and discretionary. Another niche strategy that does particularly well when equity markets collapse, unsurprisingly, is dedicated short selling; however, it is linear in its risk and as such is not considered to contain embedded optionality. Also, given its negative absolute performance when equity markets are strong, it probably has more in common with costly tail risk hedging solutions than with global macro strategies.

2.2. Exposure to non-randomness

Another school of thought focuses on implicit exposures of different hedge fund strategies to different patterns of non-randomness in returns. Typically, analysts make a distinction between trend-following or momentum-based strategies and mean-reversion or value-based strategies. Conceptually, the former are often mapped onto long vol strategies like CTAs and global macro, while the latter are mapped onto various arbitrage-type short vol strategies. Based on this distinction, Chung, Rosenberg and Tomeo (2004) propose to view different strategies as either divergent or convergent and to anchor all hedge fund allocation decisions in an optimal combination of the two. Their arguments and findings are summarised on the next page.

Convergent strategies perform best in normal times, when fundamentals prevail and investors act rationally. Expecting mean reversion, traders act on mispricing and deviations from fair value, arbitraging away inefficiencies and profiting handsomely in the process. Divergent strategies, on the other hand, perform best in turbulent times, when uncertainty prevails and investor psychology tramples fundamentals. Instead of reverting back to intrinsic
value, price deviations develop into full-fledged and well pronounced trends. Traders who successfully anticipate and ride these trends deliver outstanding performance, both in absolute terms and relative to convergent strategies.

The authors present empirical evidence: using both HFR and Credit Suisse hedge fund sub-indices, they construct proxy portfolios for convergent strategies (1/3 Equity Market Neutral + 1/3 Relative Value + 1/3 Event Driven) and divergent strategies (1/2 Managed Futures + 1/2 Global Macro). They compare and contrast their risk-return parameters, including higher moments, Sharpe and Sortino ratios, and relative performance during times of increased equity market volatility. The authors also construct and analyze five combined portfolios, increasing the allocation to divergent strategies in 20% increments from one-fifth to four-fifths of the overall portfolio. Their key findings are as follows:

- Long run returns of both strategies are relatively similar.
- Convergent strategies have lower volatility, more negative skewness, and larger kurtosis.
- Divergent strategies tend to outperform when equity market volatility spikes.
- A combined portfolio can reduce negative outliers and provide more stable returns.

2.3. Exposure to non-linearity

The third school of thought focuses on two different types of non-linear payoffs: convexity and concavity. As mentioned earlier, this goes back to Perold and Sharpe (1988), who demonstrated how different portfolio rebalancing rules resulted in two different shapes of returns. Compared to a static buy-and-hold strategy, which is linear, rebalancing back to the target asset mix is a concave strategy: as the underlying risk asset moves further and further away in either direction, the underperformance of the constant-mix strategy accelerates. Conversely, a dynamic CPPI-type strategy which increasingly buys on the way up and sells on the way down is convex: during strong and persistent market moves in either direction its outperformance accelerates.

The authors also provide a very important theoretical insight into convex and concave strategies in the context of a general equilibrium: it is effectively a zero-sum game. In other words, investors in any market cannot all enjoy convex or concave payoffs at the same time, since for every buyer there must be a seller. In aggregate, all convex and concave strategies in a particular market collectively produce a buy-and-hold portfolio. However, at any point in time there may be imbalances in supply and demand, such that one type of non-linear payoff
becomes more popular than the other. This results not only in higher implementation costs for the more popular strategy, it potentially also distorts the market. All else being equal, if convex strategies are in higher demand markets can become relatively more volatile. Conversely, if more investors switch to concave strategies, markets can become too stable. Arguably, such unhealthy stability was part of the problem during the years of “Great Moderation” leading up to the global financial crisis of 2007-08.

The concept of concave and convex strategies has been further refined and developed in the context of options markets, championed by Nassim Taleb of Black Swan fame. Originally discussed in Taleb (1997), his approach and methods were formalized and broadened in Taleb (2011a, 2011b), where the author introduced the concepts of “fragility” and “anti-fragility,” equating them with concavity and convexity, respectively.

The intuition behind these two mathematical functions is simple: a portfolio is “fragile” or concave if it suffers more from a large negative move than it gains from a similarly large positive move in the underlying market. The opposite situation occurs when a portfolio exhibits convexity, gaining more from a positive shock than losing from a negative shock. But what is the opposite of “fragile?” Taleb makes a point of distinguishing between “robust”and “anti-fragile:” whereas the former indicates an ability to merely withstand shocks and sudden changes, the latter describes a tendency to positively thrive and prosper from them. An even starker illustration...
of the contrast between convex and concave exposures can be gained by comparing payoffs from long and short straddle positions: while a large move in the underlying asset in either direction leads to ever larger and accelerating losses in a short straddle position, it conversely results in ever larger and accelerating gains in the long straddle position.

In options markets, convexity comes from being long gamma, which typically corresponds to being long volatility. Conversely, a fragile or concave position typically results from being short gamma, which in turn corresponds to a short volatility position. In an effort to popularize and explain this concept to a broad non-specialist audience, Triana (2011) described what he called the “concavity-convexity duo” as follows:

“What is a convex play? Simply put, it is one where you risk a penny to make a million. Your potential losses are limited and typically known beforehand while your potential gains are immense and unknowable. You gain disproportionately from turbulent developments in the asset market of your choice... The opposite of a convex punt is a concave one, where you risk a million to make a penny. Your potential gains are limited and typically known in advance, while your potential setbacks are huge and uncertain.”

Given that many hedge fund strategies are viewed as economically equivalent to being long or short options, it is possible to view their expected returns in terms of convex or concave payoffs. While it is not surprising that after the recent financial crisis convexity-seeking investors have in the first instance turned their attention to the relatively new phenomenon of specialist tail risk hedge funds, we believe that a number of traditional hedge fund strategies with much longer “live” track records, such as CTAs and systematic and discretionary global macro, deserve to be included in these deliberations. Whether they are viewed as a substitute or a complement to tail risk hedging will depend on a number of factors, not least the trade-off between their degree of convexity and associated costs.

3. Practical considerations and challenges
As one reviews the academic literature and discusses the above types of portfolio return drivers with market practitioners, one is tempted to think that the three schools of thought map neatly onto each other and can be invoked interchangeably in the following way:

Exhibit 6 Payoffs to Long and Short Straddles
Source: Author
However, in reality it is not quite so straightforward and simple. While in many cases there will be significant overlaps, in other circumstances there will be important and material differences. Below we illustrate this point with three specific examples.

### 3.1. Volatility exposure of CTAs

Malek and Dobrovolsky (2009) provide a rigorous and convincing explanation of why, despite popular perceptions and authoritative past research, trend-following CTA programs should not be viewed as being long volatility. While they acknowledge that there are similarities in payoff profiles between CTAs and look-back straddles, they argue that it would be inaccurate and misleading to suggest “that CTAs really have high volatility dependence or that CTAs can profit purely from volatility changes like an option strategy.” In option terms, the similarities come from similar long gamma or convexity exposures, not similar long vega or volatility exposures.

Typically, market crashes coincide with spikes in volatility, so it may appear that CTAs cash in due to their assumed long volatility exposure, but in reality they make money by riding strong and persistent trends resulting from such crashes. In contrast, when volatility spikes in directionless and range-bound markets, CTAs tend to lose money, as they get whipsawed by false breakout signals. Therefore, a meaningful allocation to trend-following CTAs can help an equity-centric institutional portfolio not because of their long volatility exposure, but because of their long gamma exposure, which builds in convexity in the domain of large equity losses.

### 3.2. Institutional rebalancing policies and CTAs

Today, most institutional investors maintain a disciplined constant-mix rebalancing policy. Yet going back to Perold and Sharpe (1988), this means that they are automatically exposed to a concave payoff relative to a buy-and-hold portfolio. This strategy outperforms in volatile and mean-reverting markets, but underperforms in quiet trending markets. In this particular case, the three schools of thought discussed earlier do not map neatly onto each other: the nature of the rebalancing strategy is concave and convergent, yet it is not short but long volatility.

Incidentally, a meaningful allocation to trend-following CTAs would be particularly well suited to such a rebalancing strategy. In both volatility-spiking scenarios mentioned above, the portfolio would have a source of positive returns: in directionless and range-bound markets, which are bad for CTAs, constant-mix rebalancing would save the day; whereas in long downward trends, that are bad for concave rebalancing strategies, CTAs would come out on top.

### 3.3. Divergence versus convexity

Divergent strategies, by definition, profit from strong market moves away from intrinsic value and equilibrium. As such, they are momentum-based and trend-following in nature. While in their analysis, Chung, Rosenberg, and Tomeo (2004) included CTAs and global macro sub-indices in this category, in reality one would be hard-pressed to find funds that are so pure in their style as to be focused exclusively on momentum and trend-following. Many global macro managers have in their standard toolkits carry trades, spread trades, and other relative value-type investments, which would be classified as convergent strategies negatively impacted by volatility. By the same token, over the years, CTAs have been increasingly adding non-trend-following modules to their programmes, such as mean-reversion trades based on overbought / oversold indicators and other technical signals. This makes perfect business sense for the hedge fund managers in question, as they build up skills and capabilities to...
achieve better diversification and more reliable “all-weather” returns. And yet, many, though certainly not all, CTAs and global macro managers still tend to exhibit highly desirable convexity in the domain of large equity losses. How can this be possible?

Our contention is that it is neither the nature of their strategy on the “convergence / divergence” criterion, nor whether they are long or short volatility, that shapes their convex return profile. Much more important, in our view, is the combination of flexibility, liquidity, and strict stop-loss discipline (“cut your losses soon, but let your profits ride”) that eventually produces the convex payoff in question. Mallaby (2010) relays how the legendary global macro trader Paul Tudor Jones explained his method:

“When you take an initial position, you have no idea if you are right… [The method is] to write a script for the market, setting out how it might behave; and then to test the hypothesis repeatedly with low-risk bets, hoping to catch the moment when [the] script [has become] reality…”

We believe it is this methodology, applied to the deepest and most liquid macro markets and combined with strict stop-loss discipline, which creates the asymmetry and convexity of returns typical of many best-in-class CTAs and global macro managers. However, it is critical to remember that hedge funds are not a homogenous asset class like equity or bonds: there is considerable dispersion of returns, especially in the global macro space, with managers often changing their style and approach over time. This is why, in our view, partnering with experienced global macro allocators is so important for institutional investors who want to identify and maintain the desired convex exposures over time.

4. Preliminary conclusions and next steps

The three schools of thought described have solid grounding in academic research and broad acceptance among market practitioners. Each provides a useful lens through which to analyze and interpret the behaviour of individual investment strategies, in both normal and turbulent markets. Combining the three can help a hedge fund analyst identify and explain the unique risk-return “signature” of a particular hedge fund style. And interestingly, all three appear to have the same message for those investors who want to strengthen their fragile portfolios: increase allocation to systematic and discretionary global macro strategies.

However, one must be careful in applying these methodologies: what is meaningful for one strategy may turn out to be misleading for another. As discussed, contrary to a widely held belief, strategies like trend-following CTAs may have ambivalent exposure to volatility. Another example would be a strategy in the options market that maintains zero net exposure to volatility, while benefitting from the volatility of volatility. In both cases, it is not long vega exposure, but long gamma exposure – in other words, not volatility but convexity – that matters. Similarly, irrespective of whether a hedge fund deploys convergent or divergent strategies, it can still exhibit that all-important asymmetry and convexity of returns that investors seek during times of crises and market dislocations. Therefore, going back to our original question of what drives portfolio returns in times of stress and how institutional investors can use that knowledge to strengthen their portfolios, we propose to start by looking at the degree and price of convexity available through various financial instruments and funds. In the context of a hedge fund allocation, we are particularly interested in how well it can support the portfolio when the main equity growth engine stalls; in other words, the degree of convexity is particularly relevant in the domain of large equity losses. As for the price of convexity, we propose to define it in terms of how much it detracts from the long-run expected returns of a classic growth-optimal portfolio.

In future research, we shall look at unique risk-return “signatures” of different hedge fund strategies, with a
particular focus on global macro and tail risk funds, to help us identify the best candidates for a "convexity completion portfolio." We shall also consider one possible way of constructing such a portfolio, given a pre-existing hedge fund allocation by a hypothetical institutional investor.

References


While it is undeniable that costs associated with tail risk hedges act as a drag on expected returns, Bhansali & Davis (2010a) propose an interesting new argument: having eliminated left tail risk, an investor can theoretically construct a much riskier portfolio – say, instead of 60/40 opt for an 80/20 equity and bond allocation. If the resulting increase in long-run expected returns is comparable to the long-run expected costs of tail risk hedges, then the investor might just be able to avoid trading off growth optimality for downside protection over the long term. Bhansali & Davis (2010b) also propose an active, rule-based monetisation approach to tail risk hedges, using the proceeds to opportunistically buy underlying equity at distressed prices. Thus, they put the traditional logic of tail risk hedging on its head: buy tail risk protection to earn more, not less equity risk premium, and to extract more, not less volatility premium, by acting like a contrarian value investor with high risk tolerance. Currently, we are not aware of any investors who have implemented this approach; until there is conclusive empirical evidence supporting this claim, we prefer to reserve judgment.


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